CCLS 6.RP.2

# Lesson 2Part 1: Introduction &UnderstandUnit Rate

## How are ratios, rates, and unit rates related?

**Ratios, rates,** and **unit rates** are all comparisons. They compare one quantity to another quantity.

A ratio compares any two quantities.

Yolanda uses 4 cups of nuts and 2 cups of dried fruit to make trail mix.

You can use a tape diagram to show this comparison.

nuts	
dried fruit	

The ratio is 4 cups to 2 cups or 4:2. Notice that the quantity of nuts is double the quantity of dried fruit.

<b>Q</b> Think	Every ratio has a related rate.

nuts	
dried fruit	

A related **rate** is an equivalent ratio that compares the first quantity in a ratio to only one of the second quantity. In this example, you know that the amount of nuts is double the amount of dried fruit. So, what if you wanted the same kind of mix but only used 1 cup of dried fruit? How many cups of nuts would you use?

Think: 4:2 is the same as \_\_\_\_\_:1?

Think: What number is 1 doubled?

The rate is 2 cups of nuts to 1 cup of dried fruit. You can also say, the rate is 2 cups of nuts per cup of fruit.

#### **Q** Think Every rate has a related unit rate.

The **unit rate** is the part of the rate that is being compared to 1. In the previous problem, the unit rate of nuts to fruit is 2. Let's look at another example.

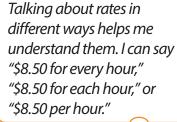
Marco earned \$85 for 10 hours of work.

Ratio of dollars to hours: 85 to 10

Rate of dollars to 1 hour: Marco earned \$85 in 10 hours, so he earned  $85 \div 10$  in 1 hour. He earned \$8.50 for each 1 hour, or \$8.50 per 1 hour.

Unit Rate: The number part of the rate 8.50 dollars per hour is 8.50.

Marco earned \$8.50 for each hour that he worked.





### 🔪 Reflect

What is the difference between a ratio and its related rate and unit rate?

## **Q** Explore It

#### A double number line can be used to find rate and unit rate.

7 What is the unit rate of miles to gallons? \_\_\_\_\_

A car can travel 300 miles on 10 gallons of gas. The ratio is 300 miles to 10 gallons.

	Miles 0 30 150 300
	Gallons 0 1 5 10
2	What do the 300 and 10 in the diagram represent?
3	How many gallons does each section along the bottom number line represent? Fill in the remaining numbers on the bottom number line.
4	Look at the corresponding pairs of numbers on the bottom and top number lines. Write a multiplication sentence to show how 10 gallons and 300 miles are related. How are 5 gallons and 150 miles related? How are 1 gallon and 30 miles related?
5	Use words to describe the relationship between the number of miles and each corresponding number of gallons.
6	Fill in the remaining numbers on the number line. What is the rate of miles per gallon for this car?

	lve the problems below as a group. Look at the model on the previous page. What pattern do you see in the numbers of miles?
	What pattern do you see in the number of gallons?
9	Now look at all of the corresponding numbers of miles and gallons. Describe the pattern
0	Write the ratio given in the problem Divide both quantities in the ratio by the second quantity in the ratio and write a new ratio. What is this ratio called? Explain
1	Look at your answer to Number 10. How can you find the related rate for a ratio?
ŗy	/ It Another Way
Vo	ork with your group to use equivalent fractions to find the rate and unit rate.
2	A 10-pound box of apples costs \$12.50. Write the ratio of cost to number of pounds as a fraction. Then find an equivalent fraction with a denominator of 1. Write the rate and unit rate to describe the cost of the apples.
3	A driver traveled 260 miles on the highway for 4 hours, driving at the same speed for
2	the whole trip. Write the ratio of miles to hours. Then use what you know about

#### Connect It

#### Talk through these problems as a class, then write your answers below.

14 Identify: Write the letter of the rate that matches each ratio.

\$7.50 : 3 pounds \_\_\_\_\_ \$3.75 to 5 pounds \_\_\_\_\_ \$6.00 : 4 pounds \_\_\_\_\_ \$13.50 to 6 pounds \_\_\_\_\_

a. \$0.75 for every 1 pound b. \$2.25 for each 1 pound c. \$2.50 for every 1 pound d. \$1.50 per 1 pound

15 Analyze: Use the information on this nutrition label to write the unit rates described below. Show your work.

#### **Q** Put It Together

17 A recipe uses 3 cups of flour and 2 cups of sugar. A Write the ratio of cups of flour to cups of sugar as a fraction: \_\_\_\_\_\_ cups. Divide the first quantity by the second quantity to find the related rate: \_\_\_\_\_ cups  $\div$  \_\_\_\_\_ cups = \_\_\_\_\_ cups to \_\_\_\_\_ cup. The unit rate is \_\_\_\_\_ . B Now write a ratio of cups of sugar to cups of flour as a fraction: \_\_\_\_\_ cups. Divide the first quantity by the second quantity to find the related rate:  $\_$  cups  $\div$   $\_$  cups =  $\_$  cups to  $\_$  cup. The unit rate is \_\_\_\_\_. C Imagine that the recipe is doubled and that 4 cups of sugar are used. Use the unit rate in A to find how much flour is needed if the recipe is made with 4 cups of sugar. Show your work. D Imagine that 6 cups of flour are used to make the recipe. Use the unit rate in B to find how much sugar is needed if the recipe is made with 6 cups of flour. E Compare your answers to C and D and explain how the two unit rates are related.

## **Understand Unit Rate**

#### **LESSON OBJECTIVES**

- Understand the concept of a unit rate.
- Use rate and unit rate language.
- Find rates and unit rate.

#### **PREREQUISITE SKILLS**

- Understand ratio concepts from 6.RP.1.
- Simplify fractions.
- Communicate relationships between two quantities using ratio notation.

#### VOCABULARY

ratio: a way to compare two different quantities

**rate:** an equivalent ratio that compares the first quantity in a ratio to only one of the second quantity

**unit rate:** the part of the rate that is being compared to 1

#### THE LEARNING PROGRESSION

Some educators use *ratio* to refer to a comparison of quantities with the same unit (green apples : apples, for example) and *rate* for comparisons of quantities with different units (revolutions : minute). The Common Core uses *ratio* to include both types of comparison. In the previous lesson, students were introduced to ratios by using ratio language to compare quantities. In this lesson students learn about rates and unit rates. They draw on their knowledge of ratios and on their fraction skills to find unit rates from ratios. Throughout this unit, students will use ratios, rates, and unit rates to solve real world and mathematical problems.

In future grades students will compute unit rates involving complex fractions. They will also reason about proportional relationships and use them to solve real world problems.

Ready Toolbox		Teacher-Toolbox.com
	Prerequisite Skills	6.RP.2
Ready Lessons	$\checkmark$	$\checkmark$
<b>Tools for Instruction</b>	1	$\checkmark$
Interactive Tutorials		<i>\</i>

#### **CCLS Focus**

**6.RP.2** Understand the concept of a unit rate  $\frac{a}{b}$  associated with a ratio *a*:*b* with  $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $\frac{3}{4}$  cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."

ADDITIONAL STANDARDS 6.RP.3, 6.RP.1 (see page A32 for full text)

STANDARDS FOR MATHEMATICAL PRACTICE: SMP 2, 6, 7 (see page A9 for full text)



Students explore comparisons of one quantity to another using ratios and related rates.

#### **STEP BY STEP**

- Introduce the Question at the top of the page.
- Discuss how the tape diagram represents the question. Note that the top of the tape represents cups of nuts and the bottom represents cups of dried fruit.
- Read the information about the ratio, rate, and unit rate as a class. Discuss that while they are all related, each is slightly different from the other two. Have students describe the differences. Be sure they see that the rate is stated in terms of 1 cup instead of 2 cups. The unit rate is like a summarized form of the rate. It includes only a number to describe how many for each 1. The "for each 1" is assumed and not written.
- Read Think as a class. Have students explain how the tape diagram shows the comparison. Discuss the meaning of rate and how rate is related to ratio.

**SMP Tip:** The terms ratio, rate, and unit rate have very specific meanings in mathematics. As you discuss each one, note that it is important to attend to precision (*SMP 6*) when using the terms.

## Hands-On Activity

#### Illustrate the rates 2:1, 3:1, and 4:1.

**Materials:** sticky dots, drawing paper, crayons or markers

- Discuss the number of wheels on a bicycle, a tricycle, and a car. Write each as a rate of wheels : vehicle.
- Give each student a sheet of sticky dots and drawing paper. On the top of the paper, students should sketch several bicycles using sticky dots for the wheels. Underneath, they should record the ratio and the rate.

\_\_\_\_wheels : \_\_\_\_bicycles = 2 wheels : 1 bicycle

• Repeat the activity for tricycles and for cars. Under each illustration they should show that the ratio and the rate are equivalent.



## **Mathematical Discourse**

• We use ratios all the time. In sports, we compare goals made to attempts. In school, we compare the number of teachers to students. What are other examples of ratios?

Responses may include other sports statistics, ratios for mixing ingredients, graduation rates.

• In the trail mix problem, how can you use a bar model to find the rate?

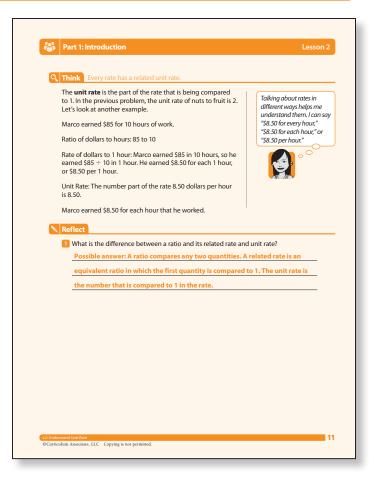
Responses may include that the bar model is a visual way to show the comparison. You see how many rectangles on the top it takes to be the same length as one rectangle on the bottom.

#### Lesson 2

Students explore rates and related unit rates. They also review the terms *ratio*, *rate*, and *unit rate*.

#### **STEP BY STEP**

- Read the problem at the top of the page with the class. Point out that it is the problem from Think on the facing page.
- Stress the difference between a ratio and a related rate. Challenge students to give examples of ratios and their related rates. (Examples: 10 passengers : 2 vans, 5 passengers : 1 van; 36 seats : 4 rows and 9 seats : 1 row)
- Read Think as a class. Use the terms *ratio*, *rate*, and *unit rate* in the context of the problem.
- Note the difference between the ratios in Think on this page and those in Think on the previous page. On this page the ratio compares two different units, dollars to hours, and on the previous page the ratio compares the same units, cups to cups.
- Have students read and reply to the Reflect directive.



## **ELL Support**

Make a table of ways to talk about a rate. Encourage students to look for these words in problems involving rates. Point out that sometimes we don't say the 1 when we say *for every, for each*, or *per*, e.g., 2 cups of nuts for each cup of fruit. Continue the table using other examples such as 4 legs and 1 table or 25 miles and 1 hour.

2 cups of nuts	to	<b>1</b> cup of fruit
2 cups of nuts	for every	<b>1</b> cup of fruit
2 cups of nuts	for each	<b>1</b> cup of fruit
2 cups of nuts	per	<b>1</b> cup of fruit

## **Mathematical Discourse**

• Explain why the ratio and the rate describing a situation have to be equivalent.

Students should see that they are equivalent since they describe the same situation. The rate is the ratio simplified so that the denominator is 1.

• Suppose a school has 800 students and 40 teachers. What is the ratio of students to teachers, the rate, and the unit rate?

800 students to 40 teachers; 20 students to 1 teacher; 20

• When might it be useful to give the ratio? The rate? The ratio shows the size of the school as well. The rate is easier to understand and simpler to use.

Students use a double number line to find the rate and unit rate when given a ratio.

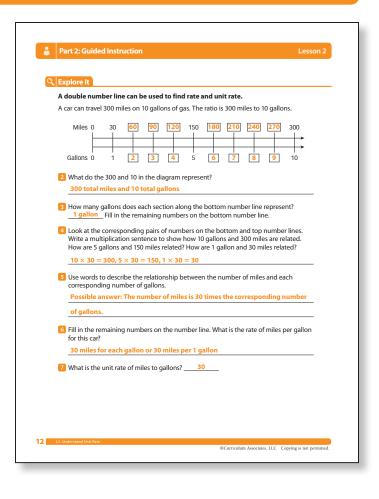
#### **STEP BY STEP**

- Tell students that they will have time to work individually on the Explore It problems on this page and then share their responses in groups. You may choose to work through the first problem together as a class.
- As students work individually, circulate among them. This is an opportunity to assess student understanding and address student misconceptions. Use the Mathematical Discourse questions to engage student thinking.
- If students do not understand how to label the top number line, ask them to find the number of miles that corresponds to one gallon. Then discuss how they can use the ratio 30:1 to complete the number line.
- Take note of students who are still having difficulty and wait to see if their understanding progresses as they work in their groups during the next part of the lesson.

## **Visual Model**

#### Use a model to find rate and unit rate.

- Draw an oval or other symbol for a gallon on the board. Have students read the Explore It problem and tell you how many gallon symbols you should draw in all.
- Ask how many miles are given in the problem. Say that the car travels the same distance on each gallon. Have students figure out how far the car travels on one gallon.
- Write 30 in each of the gallon symbols.
- Use the visual model to reinforce the relationship shown by the double number line.



Lesson 2

## **Mathematical Discourse**

• We have used three different visual models for ratios—a bar model, tape diagram, and double number line. What is one way all are alike?

Responses may include that all show the ratio and can also be used to figure out the rate and unit rate.

• How are they different?

Students may see that the bar model and number line are aligned so it is easy to see the unit rate. The tape diagram is for comparisons of the same unit, so the rectangles are the same size.

• Is there one model that helps you see the relationship more clearly? What do you like about it?

Students' explanations should involve explaining which model helps them see the relationship more clearly.

#### Part 2: Guided Instruction

#### AT A GLANCE

Students work with the double number line to reinforce the concepts of unit rate and rate in the context of the problem on page 12. They also use equivalent fractions to find rate and unit rate.

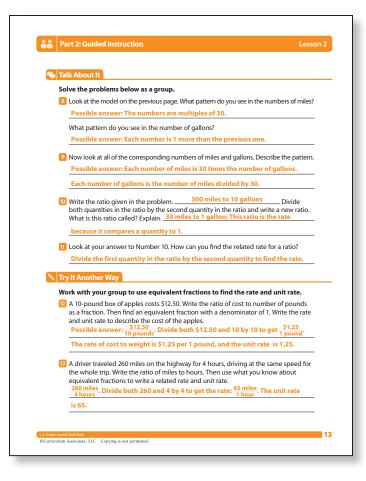
#### **STEP BY STEP**

- Organize students into pairs or groups. You may choose to work through the first Talk About It problem together as a class.
- Walk around to each group, listen to, and join in on discussions at different points. Use the Mathematical Discourse questions to help support or extend students' thinking.
- Note that there is often more than one way to describe a pattern. To make sure a pattern is valid, have students check to see if it is true for all the numbers on the double number line.
- Direct the group's attention to Try It Another Way. Have a volunteer from each group come to the board to explain the group's solutions to problems 12 and 13.

**SMP Tip:** Students look for and make use of structure (*SMP* 7) as they describe the patterns found on the double number line and in the related ratios. Occasionally ask them to describe the numeric patterns in other diagrams and ratio tables they are working with.

#### STUDENT MISCONCEPTION ALERT: Some

students may reverse the two quantities. Have students read the problem and tell what it is asking for. Note that the word order is important because it tells what order the numbers should follow. It is sometimes helpful for the student to write the ratio as words before using the numbers in the problem.



## **Mathematical Discourse**

• How can you use the pattern in numbers of miles and gallons to help you write the ratio or the rate?

Responses should indicate that the pattern tells how many miles can be driven with each gallon of gas, which is the same as the rate.

• Will the rate always be a whole number? Explain or give an example.

Listen for responses that show the rate can be a fraction. For example, the associated rate for 7 miles in 2 hours is  $3\frac{1}{2}$  miles for every 1 mile. The rate for 2 cups flour for 18 muffins is  $\frac{1}{9}$  cup per muffin.



Students demonstrate their understanding of ratios, rates, and unit rates as they talk through three problems.

#### **STEP BY STEP**

Discuss each Connect It problem as a class using the discussion points outlined below.

#### **Identify:**

- Have students explain why they would divide to find the unit rate.
- After students identify the correct rate for each ratio, ask, *How can you use multiplication to check your answers*? [Multiply the rate by the number of pounds in the original ratio.]

#### Create:

- Have students read the nutrition facts. Point out that the information is given for 2 crackers.
- Ask, If you know a quantity for 2 crackers, how do you find the related quantity for 1 cracker? [divide by 2]

#### **Compare:**

- The third problem focuses on using the unit rate to compare two ratios with different second quantities.
- Have students explain why they cannot compare the hourly earnings using the information as given.
- Have students explain why finding the rate is important in this problem.

**SMP Tip:** Students reason abstractly and quantitatively (*SMP 2*) as they compare two ratios with different second numbers by finding the related rates. They understand the meaning of the quantities in context and then work abstractly to calculate the related rates. They contextualize the rates they derived to compare the girls' earnings.

-	hese problems as a class, then write your answers below. rite the letter of the rate that matches each ratio.
lk through tl	rite the letter of the rate that matches each ratio.
-	rite the letter of the rate that matches each ratio.
ldentify: Wi	
	\$7.50 : 3 pounds a. \$0.75 for every 1 pound
	\$3.75 to 5 pounds b. \$2.25 for each 1 pound
	\$6.00 : 4 pounds <u>d</u> c. \$2.50 for every 1 pound \$13.50 to 6 pounds <u>b</u> d. \$1.50 per 1 pound
	e the information on this nutrition label to write the unit rates described
below. Show	v your work.
	NUTRITION FACTS
	Serving Size 2 Crackers (14 grams) Servings Per Container: About 20
	Amount Per Serving
	Calories 50 Calories From Fat 15
There are	25 calories in 1 cracker.
One cracker	has a mass of7 grams.
There are	7.5 or 7 <sup>1</sup> / <sub>2</sub> fat calories in 1 cracker.
Possible st	udent work: 50 calories to 2 crackers = 25 calories in 1 cracker;
14 grams t	to 2 crackers = 7 grams for 1 cracker; 15 fat calories to 2 crackers =
$7\frac{1}{2}$ fat calo	ories for 1 cracker.
	Dawn earned \$97.50 for 10 hours of work. Amy earned \$120 for 12 hours of nuch did each person earn per hour? How can you use this information to
	eir earnings?
	nswer: If Dawn earned \$97.50 for 10 hours of work, she earned
\$97.50 ÷ 1	0, or \$9.75 per hour. If Amy earned \$120 for 12 hours of work, she
earned \$12	20 $\div$ 12, or \$10 per hour. You can compare these rates to find that
Amy earns	more money per hour than Dawn does.



Students work with the ratios of two ingredients to change the quantities needed when using a recipe.

#### **STEP BY STEP**

- Direct students to complete the Put It Together task on their own.
- Remind students that unit rates can be fractions and that fractional unit rates are found and used the same way as whole number unit rates are used.
- As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.
- If time permits, have students share the reasoning behind their solutions.

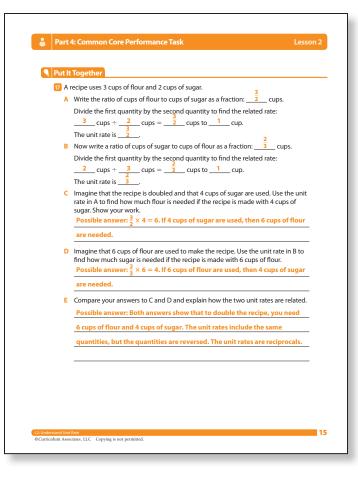
#### **SCORING RUBRICS**

See student facsimile page for possible student answers.

Α	Points	Expectations
	2	The response shows the correct ratio, rate, and unit rate.
	1	The response shows either the correct ratio or the correct rate and unit rate, but not all three.
	0	Incorrect response, or none given

B	Points	Expectations
	2	The response shows the correct ratio, rate, and unit rate.
	1	The response shows either the correct ratio or the correct rate and unit rate, but not all three.
	0	Incorrect response, or none given

С	Points	Expectations
	2	Student demonstrates the correct answer by multiplying by the unit rate, making a double number line, or using a table.
	1	Student's work shows some evidence of proportional reasoning but is not well developed or contains a minor error.
	0	Incorrect response, or none given



D	Points	Expectations
	2	Student demonstrates the correct answer by multiplying by the unit rate, making a double number line, or using a table.
	1	Student's work shows some evidence of proportional reasoning but is not well developed or contains a minor error.
	0	Incorrect response, or none given
F		

Points	Expectations
2	The response indicates the student understands that the rates show an inverse relationship and some explanation of the relationship is given.
1	The response uses a word such as <i>reciprocal</i> but does not explain the relationship in depth.
0	The response does not recognize that the rates show an inverse relationship or no response is given.

## **Intervention Activity**

# Use concrete materials to find and apply a unit rate.

Materials: paper cups, paper clips, paper, pencils

Have students work in groups of 2 or 3. Give each group 4 cups and 24 paper clips. Have students find and record the ratio of paper clips to cups. Then have them place the paper clips into cups so that each cup contains the same number. Have them record the ratio of paper clips to one cup. Then have students find and record the ratio of paper clips to 2 cups and to 3 cups.

Ask students to identify and record the unit rate. Then have them explain how they could use the unit rate to find other ratios with the same unit rate without having to count the clips in the cups.

## **On-Level Activity**

# Draw and use a double number line to find a unit price.

Tell students that a package of 12 markers costs \$3.00.

Have them draw a double number line with the top line labeled *Cost* and the bottom *Number of Markers*. Ask them how many vertical lines they would need to show all 12 markers. Have them draw the lines and label the bottom number line. Discuss how to find the cost of 1 marker. Emphasize that the cost of 1 marker is called the unit rate. Once students have found the unit rate, have them use it to number the top number line. Have students draw conclusions from the number line and share them with the class.

## **Challenge Activity**

#### Find and compare unit rates.

Materials: index cards

On the board write,

For 6 fruit baskets, Ben used 24 oranges, 6 grapefruit, 18 apples, and 3 pounds of cherries.

For 8 fruit baskets, Bill used 24 oranges, 8 grapefruit, 32 apples, and 6 pounds of cherries.

For each type of basket, have students find the unit rate for each type of fruit. Have students compare the contents of the two types of basket.

Give students an index card. On the front, the students will state the number of baskets and the amount of each type of fruit used for all the baskets. On the back, the students will list the unit rate for each type of fruit they listed on the front. Students exchange cards and find the unit rates. They check their answers with the information on the back of the card. Encourage students to create their own similar problems using different numbers of baskets and contents, and exchange cards.