In Lesson 12 you learned about solving problems involving percents. In this lesson, you will solve percent problems that involve increases and decreases. Take a look at this problem.

In May, Susana earned $40 from pet-sitting. In June, her earnings increase 200%. How much does she earn in June?

**Explore It**

Use the math you already know to help understand the problem.

- First consider 100%. What is 100% of $40? __________
- How does 200% compare to 100%? __________
- How much is 200% of $40? Explain how you know. __________
- How much is 200% more than $40? __________
- How much did Susana earn in June? __________
- Explain what it means when a value increases by 100%, and what it means when a value increases by 200%. __________
When a quantity changes over time, it is often useful to compare the original quantity and the new quantity by describing the difference, or amount of change, as a percent. The percent change is the ratio that compares the amount of the change to the original amount.

\[
\frac{\text{amount of change}}{\text{original amount}} = \text{percent change}
\]

The percent change will be a percent increase when the new amount is greater than the original amount. The percent change will be a percent decrease when the new amount is less than the original amount. You can use a ratio to find Susana’s June earnings.

\[
\frac{x}{40} = 200\%
\]

\[
\frac{x}{40} = \frac{200}{100}
\]

\[
\frac{x}{40} = 2
\]

\[
x = 80
\]

In June, Susana earned $40 + $80, or $120.

---

Percent error is the ratio describing how far an estimate is from the actual amount.

\[
\frac{\text{amount of error}}{\text{actual amount}} = \text{percent error}
\]

For example, if you estimate that your book is 11 inches long but it is really 10.875 inches long, the percent error is \( \frac{0.125}{10.875} \) or about 1.1%.

---

Reflect

1. How is a percent error like a percent change?
Read the problem below. Then explore different ways to find the percent change.

Nassim scored 5 goals in his first soccer season and 8 goals in his second soccer season. What was the percent increase in the number of goals he scored?

**Picture It**

You can use a bar model to compare the change to the original amount.

Once you know the amount of increase, you can find the percent increase.

**Model It**

You can use a proportion to compare the change to the original amount.

\[
\frac{\text{amount of change}}{\text{original amount}} = \text{percent change}
\]

\[
\frac{8 - 5}{5} = \frac{x}{100}
\]

\[
\frac{3}{5} = \frac{x}{100}
\]
Part 2: Guided Instruction

Connect It

Now you will solve the problem from the previous page and a new, similar problem.

2 Solve the proportion in the Model It on the previous page. By what percent did the number of goals Nassim scored increase?

3 Explain the relationship between the \( x \) in the proportion and the \( x \) in the bar model and tell what each \( x \) represents.

4 Nassim scored 12 goals in his third soccer season. Write and solve a proportion to show the percent increase in the number of goals scored from the first season to the third season.

5 What does it mean to have a percent increase of more than 100%?

6 Explain why it makes sense for the original amount to be the denominator when you write a proportion to find the percent change.

Try It

Use what you’ve learned to solve these problems. Show your work on a separate sheet of paper.

7 Amelia’s mom baked 48 cookies. After Amelia and her friends walk through the kitchen, there are 18 cookies left. What is the percent decrease in the number of cookies?

8 The first month Tan had his new phone, he downloaded 5 apps on it. Six months later, he has 22 apps on his phone. What is the percent increase in the number of apps Tan has on his phone?
Read the problem below. Then explore different ways to understand it.

Leo estimates that a package weighs 24 pounds. The postal clerk weighs it and finds it weighs 20 pounds. What is the percent error in Leo's estimate?

**Picture It**

You can use a bar model to help understand the problem.

```
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>actual</td>
<td>20</td>
</tr>
<tr>
<td>error</td>
<td>4</td>
</tr>
<tr>
<td>x%</td>
<td>100%</td>
</tr>
</tbody>
</table>
```

**Model It**

You can use a proportion to help understand the problem.

\[
\frac{\text{amount of error}}{\text{actual amount}} = \frac{x}{100}
\]

\[
\frac{24 - 20}{20} = \frac{x}{100}
\]

\[
\frac{4}{20} = \frac{x}{100}
\]
Connect It

Now you will solve the problem from the previous page and a new, similar problem.

9 Solve the proportion in the Model It on the previous page. What was the percent error in Leo’s estimate?

10 Explain the relationship between the $x$ in the proportion and the $x$ in the bar model and tell what each $x$ represents.

11 Leo estimated the weight of another package to be 20 pounds but the weight was actually 24 pounds. Write and solve a proportion to show this percent error. Write your answer to the nearest whole percent.

12 Both of Leo’s estimates were off by 4 pounds. Explain why the percent error was different for the two estimates even though the difference was the same.

Try It

Use what you’ve just learned to solve these problems. Show your work on a separate sheet of paper.

13 Emma needs 84 centimeters of ribbon. She measures and cuts a piece from a spool of ribbon. Later she finds out she has actually cut 80 centimeters of ribbon. What is her percent error? Write your answer to the nearest whole percent.

14 Christopher estimates it will take him half an hour to complete his math homework. He is able to complete it in 25 minutes. What is the percent error in his estimate?
Study the model below. Then solve problems 15 – 17.

When Juan got his puppy, she weighed 8 pounds. Now that she is 1 year old, her weight is 60 pounds. What is the percent increase in the puppy’s weight?

Look at how you could write a proportion that will help you solve the problem.

\[
\frac{\text{amount of change}}{\text{original weight}} = \frac{x}{100}
\]

\[
\frac{52}{8} = \frac{x}{100}
\]

\[
6.5 = \frac{x}{100}
\]

\[
100 \cdot 6.5 = \frac{x}{100} \cdot 100
\]

\[
650 = x
\]

Solution: The puppy’s weight increased 650%.

15 The tennis team is selling tickets to a car wash for $6. When they do not sell very many tickets, the team decreases the price 25%. What is the new cost of a ticket?

Show your work.
16. Irene thinks she has space for a 45-inch-wide bookcase. It turns out that she only has space for a 40-inch-wide bookcase. What is the percent error in Irene’s measurement? 

*Show your work.*

**Solution:**

17. On Thursday, 30 students went to after-school tutoring. On Friday, 6 students went. What is the percent decrease in the number of students who went to tutoring? Circle the letter of the correct answer.

A 20%
B 80%
C 400%
D 500%

Brittany chose A as the correct answer. How did she get this answer?

**Pair/Share**

If Irene thinks she has space for a 35-inch-wide bookcase, will the percent error be the same?

**What is the ratio of the amount of error to the actual width?**

**What number do you use in the denominator of the ratio?**

**Pair/Share**

Why can’t a percent decrease be greater than 100%?
Solve the problems. Mark your answers to problems 1–4 on the Answer Form to the right. Be sure to show your work.

1. Mr. Krogman usually prices the umbrellas in his store at $8 each. However, on rainy days he increases the price by 75%. How much does he charge for an umbrella on a rainy day?
   A. $2
   B. $6
   C. $9.38
   D. $14

2. The choir needs $1,000 to attend the regional competition but only has $400 in their treasury. By what percent must the choir increase their funds so that they can attend?
   A. 40%
   B. 60%
   C. 150%
   D. 250%

3. The planners of the school carnival estimate that they will sell 500 hotdogs. They only sell 400. What is the percent error in their estimate?
   A. 20%
   B. 25%
   C. 80%
   D. 100%
4. Last year 80 students signed up for a summer trip to Washington, DC. This summer 50 students have signed up to go. What is the percent decrease in the number of students?

A. 30%
B. 37.5%
C. 60%
D. 62.5%

5. Diana guesses that there are 120 gumballs in a jar. There are actually 96. In another game she guesses that there are 75 jelly beans in a jar. There were actually 60. In which game did Diana have the smallest percent error?

Show your work.

Answer ________________________________

6. In the spring, the owner of a sporting goods store decreases the price of winter gloves from $10.00 to $8.00. She increases the price of swimming goggles from $8.00 to $10.00. Without doing the math, do you think that the percent decrease in the price of gloves is the same as the percent increase of the goggles? Explain why or why not.

Answer ________________________________

Now use math to show whether or not the percent decrease and percent increase are the same. Explain why or why not.

Show your work.

Answer ________________________________

Self Check: Go back and see what you can check off on the Self Check on page 77.
Lesson 13 (Student Book pages 112–121)
Proportional Relationships

**LESSON OBJECTIVES**

- Set up and solve multi-step problems involving percent increase and decrease.
- Set up and solve multi-step problems involving percent error.

**PREREQUISITE SKILLS**

- Work fluently among fractions, decimals, and percents.
- Understand ratio, unit rate, and proportions.
- Form equivalent ratios and use equivalent ratios to solve problems.
- Find a unit rate.

**VOCABULARY**

**percent**: the number of parts per 100

**percent change**: the ratio that compares the amount of change to the original amount

**percent increase**: the percent a quantity increases from its original amount

**percent decrease**: the percent a quantity decreases from its original amount

**percent error**: the ratio that describes how far an estimate is from the actual amount

**THE LEARNING PROGRESSION**

In Grade 6, students used ratio tables and unit rates to solve problems.

In this grade, students have learned to decide whether two quantities are in a proportional relationship. They learned to identify the constant of proportionality and to represent proportional relationships by equations. In this lesson, students expand their understanding of proportional reasoning, solving multi-step ratio and percent problems by writing and solving proportions.

In Grade 8, students will graph proportional relationships, interpreting the unit rate as the slope of the graph. Students will also compare two different proportional relationships presented in different ways. They will then use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane, deriving the equation $y = mx + b$ for a line intercepting the vertical axis at $b$, and the equation $y = mx$ for a line through the origin.

**CCLS Focus**

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

**ADDITIONAL STANDARDS**: 7.RP.2.c (see page A32 for full text)

**STANDARDS FOR MATHEMATICAL PRACTICE**: SMP 1–4, 6 (see page A9 for full text)
AT A GLANCE

Students read a word problem and use their prior knowledge of percent to understand percent increase.

STEP BY STEP

• Tell students that this page helps them use what they know about percents to solve a type of problem about the percent of increase or decrease. It models building the solution to a problem one step at a time and writing to explain the solution.

• Have students read the problem at the top of the page.

• Work through Explore It as a class.

• Guide students to understand that since 200% is twice as much as 100%, the corresponding amount is also twice as much.

• Ask student pairs or groups to explain their answers for the remaining questions. Use the Mathematical Discourse questions to discuss the last question.

• Point out to students the difference between finding 200% of $40 and finding how much more $200 is than $40.

ELL Support

• Make two columns on the board. Title one column “Part” and the second column “Whole.”

• Have students decide where the phrases amount of change, original amount, percent increase, percent decrease, error in measurement, and correct measurement should be written.

• If time allows, have students brainstorm previous vocabulary words or phrases that could be added to the chart.

• Keep the chart posted and add to it when appropriate.

Mathematical Discourse

• Can you explain why an increase of 100% actually doubles the original amount?

Students should indicate that a 100% increase is the original amount plus the same amount again. Ask other students to restate the explanation in their own words.

• Can you explain why an increase of 200% actually triples the original amount?

Students should indicate that a 200% increase is the original amount plus the same amount two more times. Follow up by asking for examples of an amount and then the amount that is a 100% or 200% increase.
AT A GLANCE
Students use ratios and proportions to solve percent increase and decrease problems and percent error problems.

STEP BY STEP
• Read Find Out More as a class. Point out that, depending on the situation, the percent of change can be a percent increase or percent decrease. Discuss the new vocabulary, percent increase and percent decrease, with students.
• Remind students that a percent ratio compares the parts to the whole. For percent change, the part is the amount of change and the total is the original amount.
• Show how both ratios used in the proportion compare parts to whole, even though this time 200% is actually more than the original amount.
• Ask a volunteer to explain why $40 and $80 are added to get the final answer of $120. [The final amount is the original $40 plus the increase of $80.]
• Help students understand that percent error is just another form of a part-to-whole ratio.
• Have students independently write their answer for the Reflect question and have volunteers share their responses.

Visual Model

Use a square to visually represent a proportion.

A square is another visual representation to help write a proportion. Draw a square on the board divided into two rows and two columns. The columns represent each ratio. In the first column, write the word part in the top square and whole in the bottom square: This represents the ratio part-to-whole. In the second column, write percent in the top square and 100 in the bottom square: This represents percent-to-100. This will help students remember the form of a proportion.

Real-World Connection

Encourage students to think about everyday situations where people might encounter a percent increase or decrease, or percent error. Have volunteers share their ideas.

Examples: salary increase, increase in cost of video games, percent error in science experiments.

Find Out More

When a quantity changes over time, it is often useful to compare the original quantity and the new quantity by describing the difference, or amount of change, as a percent. The percent change is the ratio that compares the amount of the change to the original amount.

\[
\frac{\text{amount of change}}{\text{original amount}} = \text{percent change}
\]

The percent change will be a percent increase when the new amount is greater than the original amount. The percent change will be a percent decrease when the new amount is less than the original amount. You can use a ratio to find Susana's June earnings.

\[
\begin{align*}
\frac{40 - 40}{40} &= \frac{200}{100} \\
\frac{200}{40} &= \frac{x}{100} \\
x &= 500 \\
x &= 80
\end{align*}
\]

In June, Susana earned $40 plus $80, or $120.

Percent error is the ratio describing how far an estimate is from the actual amount.

\[
\frac{\text{amount of error}}{\text{actual amount}} = \text{percent error}
\]

For example, if you estimate that your book is 11 inches long but it is really 10.875 inches long, the percent error is \( \frac{0.125}{10.875} \) or about 1.1%.

Reflect

How is a percent error like a percent change?

The numerator is the difference between two quantities and the denominator is the original or actual amount.
Students use a bar model to set up a proportion to solve a percent increase problem.

**STEP BY STEP**

- Read the problem at the top of the page as a class.
- Have a volunteer explain how the bar model shows the change to the original amount.
- Help students understand that the proportion represents part-to-whole relationships that are equivalent.

**SMP Tip:** Students are demonstrating quantitative reasoning (SMP 2) when they create coherent representations of Nassim’s soccer goals. The bar model representation helps students understand the problem in order to represent the situation symbolically with a proportion.

**Concept Extension**

**An alternate way to view percent decrease or increase.**

- An alternate way to determine percent decrease or increase is to always subtract the starting value from the ending value to determine the amount of change.
- If there is a percent increase, then the difference will be positive. If there is a percent decrease, then the difference will be negative.
- For example, if Nassim scored only 2 goals in the spring season, then his amount of change would be determined by subtracting \(2 - 5\). His percent change would be \(-60\%\), indicating a percent decrease.

**Mathematical Discourse**

- **Can you explain what parts of the situation the ratio \(\frac{3}{5}\) represents?** What does 5 represent? What does the 3 represent?

  Students may be confused by the fact that the 8 in the problem is not used in the ratio. Listen for responses that relate 5 to the number of goals scored in the first season and 3 to the additional number scored in the second season.

- **Why are we looking for an equivalent ratio \(\frac{x}{100}\)?**

  Responses should explain that because we are looking for a percent, we are looking for the ratio of a number to 100.
Students revisit the problem on page 114 to solve proportions involving percent increase.

**STEP BY STEP**

- Read Connect It as a class. Be sure to point out that the questions refer to the problem on page 114.
- Watch for students who write $\frac{5}{7}$; they might believe the numerator must always be smaller than the denominator.
- Have volunteers explain why the denominator is always the whole.

**Concept Extension**

**Write proportion problems.**

- Challenge students to write a situation given a proportion.
- Write a proportion on the board.
- Have students write a percent increase, percent decrease, and a percent error word problem that could be solved with the given proportion.

**TRY IT SOLUTIONS**

7. **Solution:** 62.5%; Students may set up and solve the proportion $\frac{30}{48} = \frac{x}{100}$.

**ERROR ALERT:** Students who wrote 37.5% may have used the number of cookies left rather than the number of cookies taken.

8. **Solution:** 340%; Students may set up and solve the proportion $\frac{17}{5} = \frac{x}{100}$.
Students use a bar model to set up a proportion to solve a percent error problem.

**STEP BY STEP**

- Read the problem at the top of the page as a class.
- Have a student volunteer explain how the bar model relates to the proportion.

**SMP Tip:** Discuss with students how important it is to attend to precision (SMP 6). Percent error problems require students to determine what the given information means and how they can use it to solve the problem.

**Concept Extension**

**Apply percent error to science.**

- In science, percent error is determined by the formula:
  \[
  \text{percent error} = \left( \frac{\text{experimental value} - \text{theoretical value}}{\text{theoretical value}} \right) \times 100
  \]
- Experimental values are obtained from experimental data. Theoretical values are obtained from authoritative sources (e.g., textbooks).
- Ask students to describe how the values are alike and how they are different.

**Mathematical Discourse**

- **In your own words, how would you explain percent error?**
  
  “Percent error” describes how far off a value is from the actual answer, or how large an error is.

- **What other fractions could replace \[ \frac{3}{5} \] or \[ \frac{x}{100} \] and still mean the same thing?**

  Students should suggest that any fraction equivalent to the original fractions will work.
Students revisit the problem on page 116 to solve proportions involving percent error.

**STEP BY STEP**

- Read Connect It as a class. Be sure to point out that the questions refer to the problem on page 116.
- Continue to have students show connections between the bar model and the proportion used to solve the problem.

**ELL Support**

- Prior to the lesson, activate student prior knowledge on how to solve proportions.
- Have students solve several proportions that do not involve language and are already set up for them.
- Post a poster on the wall with a sample problem showing the steps in solving a proportion for future reference.

**TRY IT SOLUTIONS**

13  *Solution:* 5%; Students may set up and solve the proportion $\frac{4}{84} = \frac{x}{100}$. Students will need to round up to 5%.

14  *Solution:* 20%; Students may set up and solve the proportions $\frac{5}{25} = \frac{x}{100}$.

**ERROR ALERT:** Students who wrote 17% may have used 30 minutes as the actual amount.
Students use proportions to solve word problems involving percent change and percent error problems.

**STEP BY STEP**

- Ask students to solve the problems individually and write their solution in a sentence.
- When students have completed each problem, have them Pair/Share to discuss their solutions with a partner or in a group.

**SOLUTIONS**

**Ex** A proportion is shown as one way to solve the problem. Students may also use a bar model to help set up the proportion.

15 **Solution:** $4.50; Students could solve the problem by using the equation $6 - (0.25 \times 6) = c$.

16 **Solution:** 12.5%; Students could solve the problem by solving the proportion $\frac{6}{50} = \frac{x}{100}$.

17 **Solution:** B; Brittany used the ratio of Friday’s students instead of Thursday’s students.

Explain to students why the other two answer choices are not correct:

C is not correct because the ratio of $\frac{24}{6}$ was used.

D is not correct because the ratio of $\frac{30}{6}$ was used.
Students use proportions to solve percent increase, percent decrease, and percent error problems that might appear on a mathematics test.

**STEP BY STEP**

- First, tell students that they will use proportions to solve percent increase, percent decrease, and percent error problems. Then have students read the directions and answer the questions independently. Remind students to fill in the correct answer choices on the Answer Form.

- After students have completed the Common Core Practice problems, review and discuss correct answers. Have students record the number of correct answers in the box provided.

**SOLUTIONS**

1. **Solution:** D; Use the equation \(8 + (0.75 \times 8) = c\).
2. **Solution:** C; Use the proportion \(\frac{600}{400} = \frac{x}{100}\).
3. **Solution:** B; Use the proportion \(\frac{100}{400} = \frac{x}{100}\).
4. **Solution:** B; Use the proportion \(\frac{30}{80} = \frac{x}{100}\).
5. **Solution:** The percent errors are the same. Explain why or why not. The percent of change is the same, but the original amounts are different.
6. **Solution:** No, because the original amounts are different; 20% decrease; 25% increase. See possible student work above.
Assessment and Remediation

• Ask students to find the percent decrease of a bicycle’s price if the original price was $120 and the sale price is $80. [33.3%]

• For students who are struggling, use the chart below to guide remediation.

• After providing remediation, check students’ understanding. Ask students to explain their thinking while finding the percent error in this scenario: The actual temperature was 65°, but Tom read 69° on his thermometer. [6.1%]

• If a student is still having difficulty, use Ready Instruction, Level 7, Lesson 10.

<table>
<thead>
<tr>
<th>If the error is . . .</th>
<th>Students may . . .</th>
<th>To remediate . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.7%</td>
<td>have compared the sale price to the original price instead of using the $40 decrease.</td>
<td>Remind students to compare the decrease to the original amount.</td>
</tr>
<tr>
<td>50%</td>
<td>have compared the $40 decrease to the sale price.</td>
<td>Have students draw a bar model to help to show a ratio that compares the part to the whole.</td>
</tr>
<tr>
<td>300%</td>
<td>have compared the original price to the discount.</td>
<td>Remind students that a 300% decrease is not reasonable and that a ratio compares the part to the whole.</td>
</tr>
</tbody>
</table>

Hands-On Activity

Use models to solve a proportion.

Materials: one page of square grid paper for each student

Distribute a page of square grid paper to each student. Write the proportion \( \frac{3}{5} = \frac{x}{100} \) on the board. Ask students to draw a rectangle around five squares and shade in three of them to show the ratio \( \frac{3}{5} \). Have students enlarge the rectangle by adding five more squares and shade three of the new squares. Students should continue to add and shade squares until there is a total of 100 squares. Students should then count the number of shaded squares and see that there are 60 shaded squares out of 100 total squares.

Challenge Activity

Writing percent change as a multiplication problem

Percent change problems can be expressed as multiplication problems. For example, an increase of 5% on an item originally priced as $20 can be found by multiplying $20 by 1.05. Challenge students to find a pattern that helps them figure out the multiplier, given the percent change. [Possible answer: the multiplier is the percent written as a decimal plus 1.]